Reports, &c. (*continued*).

Rome:—Triplice Omaggio alla Santità di Papa Pio IX, nel suo giubileo episcopale offerto dalle tre Romane Accademie Pontificia di Archeologia, Insigne delle Belle Arti, Pontificia de' Nuovi Lincei. Scienze. 4to. *Roma* 1877.
The Accademia Pontificia de' Nuovi Lincei.

Arya Bhatta. The Elements of Plane Geometry in 48 propositions, from the Sanscrit text, edited on the principle of Euclid, by Jasoda Nandan Sircar. 12mo. *Calcutta* 1878. The Editor.

Brongniart (Ch.) Note sur un nouveau genre d'Orthoptère fossile de la famille des Phasmiens. 8vo. *Paris.* Note rectificative sur quelques Diptères tertiaires. 8vo. *Lille* 1878. The Author.

Fayrer (Sir Joseph) F.R.S. On the Bael Fruit and its Medicinal Properties and Uses. 12mo. *London* 1878. The Author.

Hooker (Sir J. D.), F.R.S. The Flora of British India. Part 5. 8vo. *London* 1878. The India Office.

Lubbock (Sir John), F.R.S. Pre-historic Times, as illustrated by Ancient Remains and the Manners and Customs of Modern Savages. Fourth edition. 8vo. *London* 1878. The Author.

Matton (L. P.) Quadrature du Cercle, son existence prouvée. 4to. *Lyon* 1878. Polysection et Polysectrices. 4to. 1878.
The Author.

Mourek (V. E.) A Dictionary of the English and Bohemian Languages. 12mo. *Prague* 1879. The Author.

Roscoe (H. E.), F.R.S., and C. Schorlemmer, F.R.S. A Treatise on Chemistry. Vol. II. Metals. Part 1. 8vo. *London* 1878.
The Authors.

February 6, 1879.

W. SPOTTISWOODE, M.A., D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

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I. "On certain Dimensional Properties of Matter in the Gaseous State. Part I. Experimental Researches on Thermal Transpiration of Gases through Porous Plates, and on the Laws of Transpiration and Impulsion, including an Experimental Proof that Gas is not a Continuous Plenum. Part II. On an Extension of the Dynamical Theory of Gas which includes the Stresses, Tangential and Normal, caused by a Varying Condition of the Gas, and affords an explanation of the Phenomena of Transpiration and Impulsion." By OSBORNE REYNOLDS, F.R.S., Professor of Engineering at Owens College. Received January 17, 1879.

Abstract of Part I (Experimental).

Section I (Introduction).

1. The motion of gases through minute channels such as capillary tubes, porous plugs, and apertures in thin plates has been the subject of much attention during the last fifty years. The experimental study of these motions, principally by Graham, resulted in the discovery of important properties of gases, and it is largely, if not mainly, as affording an explanation of these properties, that the molecular theory has obtained such general credence.

It does not appear, however, that either the experimental investigations of these motions, or the theoretical explanations of the properties revealed, have hitherto been in any sense complete. There exists a whole class of very marked phenomena which have escaped the notice of Graham and other observers, while several of the most marked and important facts discovered by Graham have hitherto remained unconnected by any theory.

2. Amongst the best known of the phenomena is the difference in the rates at which different gases transpire through minute channels, and the consequent difference in the pressure which ensues when two different gases, initially at the same pressure, are separated by a porous plate. But it does not appear that hitherto any attempt has been made to ascertain the existence of what may be considered a closely analogous phenomenon—that a difference of temperature on the two sides of the plate might cause gas, without any initial difference of pressure or any difference in chemical constitution, to pass through the plate—nor am I aware that such a result from a difference of temperature has been in any way surmised.

I have now ascertained by experiments, which will be described at length, that a difference of temperature may be a very potent cause of transpiration through porous plates. So much so, that with hydrogen on both sides of a porous plate, the pressure on the one side being that of the atmosphere, a difference of 160° F. (from 52° to 212°) in the temperature on the two sides of the plate secured a permanent difference in the pressure on the two sides equal to an inch of mercury; the higher pressure being on the hotter side. With different gases and different plates various results were obtained, which, however, as will be seen, are connected by definite laws.

3. Again, although Graham found that he obtained not only very different results, but also very different laws of motion with plates of different coarseness, or with plates and capillary tubes, neither he nor any subsequent observer appears to have followed up this lead. As regards Graham, this appears to me to be somewhat surprising. For although he may have considered the mere difference in the results to have been analogous to the difference found by Poiseuille for liquids, it would seem as though the difference in the laws of motion should have excited his curiosity; and then, as he was avowedly of opinion that gas is molecular, he could hardly have failed to observe that so long as the mean distance separating the molecules in the gas bore a fixed relation to the breadth of the openings in his plates, he should have had the same laws of motion. This view, however, appears to have escaped him as well as all subsequent observers, otherwise it would have been seen that with a simple gas such as hydrogen, similar results must be obtained so long as the density of the gas is inversely proportional to the lateral dimensions of the passages through the plates.

By experiments to be described I have now fully established this law. I find that with different plates similar results are obtained when the densities of the gas with the different plates bear certain fixed ratios, and that this is the case whatever may be the cause of transpiration, *i.e.*, a difference of temperature or a difference of pressure; a difference of gas I have not investigated, as it was obviously unnecessary to do so. Thus, with two plates, one of stucco and the other of meerschaum, similar results of transpiration caused by pressure were obtained when the densities with the two plates were respectively as 1 to 5·6 both with hydrogen and air, and at pressures ranging from 30 to 2 inches of mercury; also with the same two plates similar results of thermal transpiration were obtained when the densities were respectively as 1 to 6·5 both for hydrogen and air, and through a range of densities from 30 inches to ·25 of an inch of mercury, the discrepancy between 5·6 and 6·5 being in all probability owing to a slightly altered condition of the plates.

This correspondence of the results at corresponding densities holds although the law of motion changes. Thus, with air at 30 inches through the stucco plate, the law of motion was the same as that found by Graham for a stucco plate, while at the smallest pressure

(·25 inch) it was nearly the same as he found for graphite plates, or for apertures in thin plates.

Having established this law of corresponding results at corresponding densities, it became apparent that the results obtained with plates of different coarseness, and with the same plates, but with different densities of gas, followed a definite law. This law, which admits of symbolical expression, shows that there exists a definite relation between the results obtained, the lateral dimensions of the passages, and the density of the gas.

This law is important in reconciling results which have hitherto appeared to be discordant and as tending to complete the experimental investigation, but it has another and a more general importance.

It may not appear at first sight, but on consideration it will be seen that this law amounts to nothing less than an absolute experimental demonstration that gas possesses a heterogeneous structure—that it is not a continuous plenum of which each part into which it may be divided has the same properties as the whole.

It would appear that Graham must have had this proof, so to speak, under his eyes, and it is strange that both he and other observers have overlooked it. It seems possible, however, that they were not alive to the importance of such a demonstration. It is now so generally assumed that gas is molecular, that the weakness of the evidence on which the assumption is based and the importance of further proof are points which are apt to escape notice.

The Importance of an Experimental Demonstration that Gas Possesses Molecular Structure.

5. The idea of molecular gas does not appear to have originated from the recognition of properties in gas which were inconsistent with the idea of a continuous plenum, but from a wish to reconcile the properties of gas with the properties of other substances, or, more strictly, with some general property of matter. And the general conviction which may be said to prevail at the present time is owing to the simplicity of the assumptions on which the molecular hypothesis is based, and the completeness with which many of the properties of gas have been shown to result from this hypothesis. But it will be readily seen that however simple may be the assumptions of the kinetic theory, and however completely the properties of gases may be shown to follow from these assumptions, this is no disproof of the possibility that gas may be a continuous substance, each elementary portion of which is endowed with all the properties of the whole, and unless this is disproved there may exist doubts as to the necessity for the kinetic theory.

Any direct proof, therefore, that gas is not ultimately continuous altogether alters the position of the molecular hypothesis.

The Sufficiency of the Demonstration that Gas is not Structureless.

6. In order to prove that gas is not structureless, it is not necessary that we should be able to perceive the actual structure; we have only to find some property of a certain quantity of gas which can be shown not to be possessed by all the parts, some property which is altered by a rearrangement of the parts.

Hitherto I believe that no such property has been recognised, or, at all events, the conclusions to be drawn from such a property have not been recognised. The phenomena of transpiration, as well as those of the radiometer, depend on such properties, but these properties have not been sufficiently understood to bring out the conclusion. This conclusion, however, follows directly from the law indicated in Article 4, viz., that the results of transpiration depend on the relation between the size of the passages and the density of the gas.

The Results Deduced from Theory.

7. Although the existence of the phenomena of thermal transpiration, and the existence of the law of corresponding results at corresponding densities, have been verified by experiment, they were not so discovered; they followed from what appeared to be a successful attempt to complete the explanation which I had previously offered of the forces which result when heat is communicated from a surface to a gas,* and the phenomena of the radiometer.

Having found, what I had not at first perceived, that according to the kinetic theory the excess of pressure resulting from the communication of heat to a gas must depend on the fact of the surface from which the heat flows being of limited extent, and must follow a law depending on some relation between the mean path of a molecule and the size of the surface, it appeared that by using vanes of comparatively small size the force should be perceived at correspondingly greater pressures of gas.

On considering how this might be experimentally tested, it appeared that to obtain any result at measurable pressures the vanes would have to be very small indeed; too small almost to admit of experiment. And it was while searching for some means to obviate this difficulty that I came to perceive that if the vanes were fixed, then instead of the movement of the vanes we should have the gas moving past the vanes—a sort of inverse phenomenon; and then instead of small vanes small spaces might be allowed for the gas to pass. Whence it was at once obvious that in the porous plugs I should have the means of verifying these conclusions. I followed up the idea, and by a method which I devised of extending the dynamical theory of gases, so as to take into account the forces (tangential and normal) arising from a varying condition of molecular gas, I was able to deduce what appears to me to be a complete theory of transpiration.

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* "Proc. Roy. Soc.," vol. xxii, p. 402, and "Phil. Trans.," vol. clxvi, p. 726.

This theory appears to include all the results established by Graham, as well as the known phenomena of the radiometer, which, for the sake of shortness, I shall call the phenomena of *Impulsion.* I was also able definitely to deduce the results to be expected as regards both thermal transpiration and the law of corresponding densities for transpiration and impulsion.

Having made these deductions I commenced the experiments on transpiration, which so completely verified my theoretical deductions that I have been able to produce the theory in its original form, with some additions but without any important modification.

Moreover, having succeeded (not without some trouble) in rendering apparent the effect of differences of temperature in causing gas to move through fine apertures, I recurred to the original problem, and by suspending fibres of silk and spider lines to act as vanes, I have now succeeded in directly verifying the conclusion that the pressure of gas at which the force in the radiometer becomes apparent varies inversely as the size of the vanes. With the fibre of silk I obtained repulsion at pressures of half an atmosphere.

The Arrangement of the Paper.

8. My object in this paper is to describe the reasoning by which I was led to undertake the experiments, as well as the experiments themselves; but as the theory will be better understood after an acquaintance with the facts, I have inverted the natural order and given the experiments first. I include here, however, a somewhat full account of the results to be expected as deduced from the theory.

Then follows a statement of the laws of transpiration and impulsion as deduced from theory:—

Section II is devoted to the description of the experiments on thermal transpiration; Section III to the experiments on transpiration under pressure, and Section IV to the experiments on impulsion.

In this abstract it will not be possible to give more than a sketch of the matter contained in these sections. The numerous precautions and tests will have to be left unnoticed, and only a few of the experimental results can be given. The investigation occupied from February, 1878, till the beginning of August, every result being verified by repeated experiments.

The Apparatus for Thermal Transpiration.

This consisted principally of an instrument called a thermo-diffusiometer, of which the essential feature is two chambers, separated by a plate of porous material, means being provided for keeping the chambers at constant, but different, temperatures for many hours at a time, also for measuring the pressure of gas in the chambers, for exhausting the chambers, and for bringing the chambers into direct communication when required. The different temperatures were secured by a stream of steam on the one side, which gave a temperature of 212° F., and a stream of water on the other side, which gave a temperature constant for the time, but which ranged during the investigation from 47° in February to 70° in July.

The porous plates tried were of biscuit-ware, stucco, and meerschaum, and ranged in thickness from ·06 (1·5 millims.) to ·44 inch (11·2 millims). The pressures of gas within the chambers and the difference of pressure on the two sides of the plate were measured by mercury gauges. A special instrument used for reading the differential gauge read to the $\frac{1}{10000}$th of an inch (·0025 millim.).

Several weeks were spent on this apparatus in getting it tight, getting the gauges to work, and getting rid of the disturbing effects of moisture, before any definite results were obtained, but finally the instrument answered extremely well.

The Experiments on Thermal Transpiration.

The streams of steam and water having been kept going for several hours, long enough for the condition of temperature in the instrument to be perfectly steady, the tap which established communication between the chambers on the opposite sides of the porous plate having been open, so that the pressure in these chambers was equal, this tap was closed, so that the sole communication was through the porous plate. Any difference of pressure between these chambers was then read on the differential gauge.

Supposing that on the first reading the gas (whatever it might be) within the instrument was at the pressure of the atmosphere, a certain quantity of gas was then drawn out and the experiment repeated. This was done until the pressure within the instrument was as low as ·25 inch of mercury.

According to the theoretical deductions, it had appeared that when the sole communication between the two chambers was through the porous plate, and the gas in these chambers was at the same pressure, the difference of temperature would cause the gas to pass from the colder chamber to that which was hotter, until a certain difference of pressure was established, after which there would be no further change as long as the same difference of temperature was maintained, so that the result to be expected as giving evidence of thermal transpiration was a difference in the pressure on the two sides of the plate.

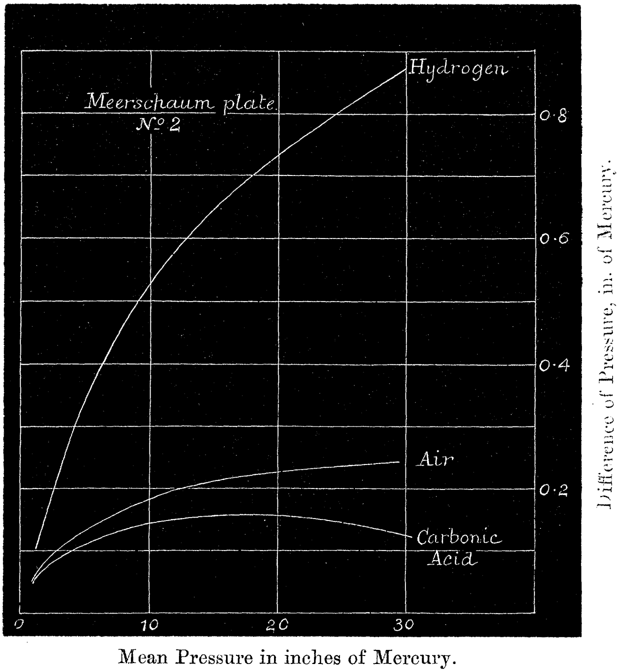
This difference was first obtained with air at the pressure of the atmosphere and a biscuit-ware plate, the difference being ·1 inch (2·54 millims).

It further appeared from the theory that the difference which would, *cæteris paribus*, depend on the difference of temperature, would also depend on the relation between the density of the gas and the coarseness of the plate, so that, *cæteris paribus*, the finer the plate the greater the difference, and this conclusion was at once verified.

A plate of meerschaum, ·25 inch (6·3 millims.) thick, gave a difference, ·25 inch with air, and ·88 with hydrogen, at the pressure of the atmosphere, while a plate of stucco of the same thickness as the meerschaum only gave a difference of ·02 inch with air and ·14 inch with hydrogen.

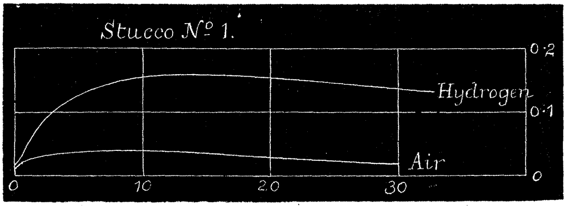
It also appeared from the theory that with the same plate and the same gas, the difference of pressure should be a maximum at some particular density, so that if the initial density was sufficient, the thermal difference of pressure would increase as exhaustion proceeded up to a certain point and then fall off, the density at which the thermal difference would be a maximum depending on the coarseness of the plate and the nature of the gas. These conclusions were verified.

FIG. 1.



For with the meerschaum plate the thermal difference for air was almost constant at pressures nearly equal to the atmosphere, but fell at an increasing rate as the density diminished (this is shown by the curve for air, fig. 1). From this it was clear that if the thermal difference reached a maximum it would be at some pressure greater than that of the atmosphere. With stucco the thermal difference for air increased as the pressure fell from that of the atmosphere, and reached a maximum only at a pressure of about 8 inches of mercury (shown in fig. 2).

FIG. 2.



A comparison of these results shows that the density at which the thermal transpiration is a maximum depends on the coarseness of the plate; and that it depends on the nature of the gas appears at once on comparing the results for hydrogen, air, and carbonic acid, which are shown on figs. 1 and 2.

Experiments with plates of various thicknesses gave the thermal difference of pressures independent of the thickness of the plate, so long as the difference of temperature was the same.

Several minor deductions from the theory were also directly verified.

The Law of Corresponding Results at Corresponding Densities.

In order to establish this law it was necessary to compare the results obtained with different plates. According to the law, the ratio of the thermal difference of pressure to the mean pressure with a particular plate and a particular gas should be the same as with another plate and the same gas, as long as the densities (or pressures) are in a fixed ratio, which is the ratio of the fineness of the plates.

A simple numerical calculation sufficed to show that this conclusion is approximately verified. On dividing the thermal differences by the mean pressure for both the meerschaum and stucco plates, it appears that the resulting numbers are approximately equal, so long as the pressure with the meerschaum is six times as great as with the stucco. This is so both for air and hydrogen, and through a range of pressures from 30 to ·35 inches with the meerschaum, and 5 to ·2 inches with the stucco.

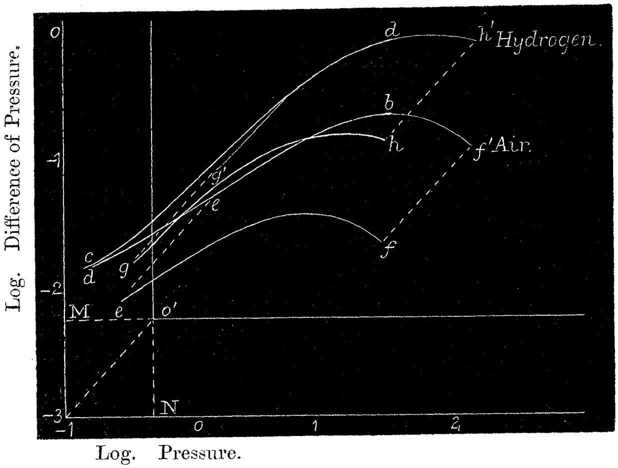
The numerical comparison does not, however, bring out the agreement in nearly as strong a light as the comparison which has been effected by a graphic method.

Graphic Method of comparing the Results.

This method consists in taking as ordinates and abscissæ, not the thermal differences and mean pressures, but the logarithms of these quantities, and the curves so formed are called the logarithmic homologues of the curves shown in figs. 1 and 2.

Any common ratio which exists between ordinates or abscissæ of corresponding points on the natural curves becomes a common difference between the ordinates or abscissæ of their logarithmic homologues, so that if the natural curves correspond after the manner that has just been described, their logarithmic homologues must be precisely similar curves, such that by shifting the one parallel to itself it can be made to fit on to the other.

FIG. 3.



In fig. 3 *a b* and *c d* are the logarithmic homologues respectively for the air curve and the hydrogen curve with meerschaum, and *e f* and *g h* are the logarithmic homologues respectively for the air and hydrogen curves with stucco. By tracing *e f* and *g h* together with their axes on the same paper, and moving the paper without turning it, until the traced curves fit the curves for meerschaum, it is found that the fit is perfect, a portion of the traced curve *e' f'* coinciding with a portion of *a b*, and a portion of *g' h'* coinciding with *c d*.

O' M and O' N, the components of the shift, are the logarithms of the ratios of the corresponding ordinates and abscissæ of the natural curves; and in the particular case to which fig. 2 refers—

$$\text{O}'\text{N} = \cdot 7 = \log. 5$$
$$\text{O}'\text{M} = \cdot 77 = \log. 5{\cdot}9$$

It is thus seen that the reason why the numerical comparison did not give absolutely consistent results is because the ratio of the corresponding abscissæ is not exactly the same as that of the corresponding ordinates, the difference being found on examination to be owing to a discrepancy in the temperatures, which affected the ratio of the corresponding ordinates, but not the ratios of the corresponding abscissæ. These, therefore, give 5 as the ratio of the coarseness of these particular plates.

The woodcut, fig. 3, merely illustrates the result. The logarithmic homologues of the curves for both air and hydrogen, plotted with the greatest care, the points marking the experiments being so close together that it was scarcely necessary to draw a curve, have been compared, and the agreement is very remarkable, the only slight deviation being that shown in fig. 3, which was found to be owing to some impurity in the hydrogen at pressures below an inch of mercury.

In order fully to appreciate the force of this agreement, it must be noticed that it is not only the portions of the curves which overlap that agree in direction, but also the distances between the curves for hydrogen and air, which are shifted in pairs.

Nothing could prove more forcibly than this fitting that the difference in the results for different plates depends on a relation between the density of the gas and the coarseness of the plates.

Experiments on Transpiration under Pressure.

According to the theoretical deductions the rate at which gas would be forced through a tube or porous plate by a difference of pressure bearing a fixed ratio to the mean pressure of the gas in passing, would vary with the mean density of the gas according to a law which would hold with different plates, the corresponding results being obtained at pressures inversely proportional to the diameters of the tubes. The differences in the laws of transpiration which Graham found with different tubes and plates are, so far as they go, in fair accordance with the law as deduced from this theory, but the range of densities over which Graham's results extend is too small to allow a very complete verification, and the chief object in these experiments was to extend this range of densities. The apparatus used was the thermo-diffusiometer, slightly modified, and without the streams of steam and water. The instrument lent itself very well to this part of the investigation. It allowed of the measurement of the time of transpiration of a definite volume of gas, measured at whatever might be the pressure of the instrument, through the porous plate, under a difference of pressure bearing a fixed ratio to the pressure within the instrument.

The times of transpiration of equal volumes of air and hydrogen through plates of stucco and meerschaum were determined at pressures varying from that of the atmosphere to a fraction of an inch of mercury.

These results, in as far as the conditions correspond, were found to agree very closely with the results obtained by Graham. Thus, through stucco at 30 inches, the comparative times of transpiration of air and hydrogen were as 2·9 to 1, Graham's results being 2·8 to 1. Through meerschaum the ratio was 3·6 to 1, Graham having found the ratio 3·8 to 1 through a graphite plate, which was in all probability finer than the meerschaum. The ratio of the times of transpiration of equal volumes of different gases, which Graham looked upon as varying only with the coarseness of the plates, was found, as was expected, to depend entirely on the relation between the pressures of the gases and the coarseness of the plates, the ratio of the times being the same as long as the pressures of the gases were inversely proportional to the coarseness of the plates.

Thus, at a pressure of 5 inches, the times for hydrogen and air through stucco, instead of being 2·9, as at the pressure of the atmosphere, were 3·6, or the same as through meerschaum at a pressure six times as great; the coarseness of the plates, as determined in the previous experiments, being 5·6. The same agreement held as long as the ratio between the pressures was maintained.

The correspondence of the results for different plates, and the complete verification of the theoretical conclusions which they afforded, is shown by comparing the logarithmic homologues of the curves in which the times of transpiration are the ordinates and the pressures the abscissæ. The fitting of the logarithmic homologues is exact, both as regards the direction of the curves and the distances between the curves, for air and hydrogen; the displacement along the abscissæ, to bring the curves into coincidence, being ·819 = log. 6·5.

As this number, 6·5, is the ratio of the coarseness of the plates, it should have corresponded with the ratio obtained by thermal transpiration, which was 5·6, with the same plates. This discrepancy, although too small to cast a doubt upon the general agreement of the results, is too large to be attributed to experimental inaccuracy, and must have been due to some change in the plates, probably arising from the plates being hot in the one case and cold in the other.

Experiments on Impulsion with a Suspended Fibre.

A single fibre of unspun silk was suspended from one end in a vertical test-tube, closed with an india-rubber cork, and connected with an air-pump, and a microscope was arranged for observing the motion of the fibre when a hot body was brought into a certain position near the test-tube.

With air in the test-tube at the pressure of the atmosphere, it was found that the fibre was carried by the air-currents towards the hot body, and this was the case as long as the pressure was greater than 8 inches of mercury, but after the tube had been exhausted below this point, the fibre moved away from the heat, and the motion steadily increased as the pressure became very small, such as $\frac{1}{50}$th of an inch of mercury.

With hydrogen in the test-tube, the fibre moved away from the heat at all pressures below that of the atmosphere, and for small pressures the motion was somewhat larger than with air.

A spider-line was also tried, and gave results similar to the fibre of silk.

It thus appeared that both with the fibre of silk and the spider-line the phenomena of impulsion were manifest at densities many hundred times greater than the highest densities at which like results are obtained with the vanes of the radiometer which are several hundred times broader than the fibre of silk. And this verifies the law of corresponding results at corresponding densities for this class of phenomena.

Abstract of Part II (Theoretical Investigation).

The characteristic as well as the novelty of this investigation is that, not only is the mean of the motions of the molecules at the point under consideration taken into account, but also the manner in which the mean motion may vary from point to point, in any direction across the point under consideration. It appears that such a variation gives rise to certain stresses in the gas (tangential and normal) and it is of these stresses that the phenomena of transpiration and impulsion afford evidence.

Instead of considering only the mean condition of the molecules comprised within an elementary volume of the gas, what is chiefly considered in this investigation is the mean condition of the molecules which cross an elementary area in a plane supposed to be drawn through the point.

Q is used to indicate a quantity belonging to a molecule such as its mass, momentum, or energy, $\sigma_x(\mathrm{Q})$ to express the rate at which Q is carried across a plane perpendicular to the direction x.

Two systems of axes are employed, xyz fixed axes, with respect to which u, v, w, are the component velocities of a molecule, and a system of axes parallel to xyz, but moving with the component velocities U, V, W, with respect to which ξ, η, ζ, have the component velocities of a molecule. U, V, W, having such values that

$$\overline{\xi^2}=\overline{\eta^2}=\overline{\zeta^2}=\frac{a^2}{2}.$$

As preliminary to the investigation, expressions are obtained for $\sigma(\mathrm{Q})$ in terms of a, U, V, and W, on the supposition that the gas is uniform. This is accomplished by the application of well-known methods.

When the condition of the gas varies from point to point, the molecules are considered as consisting of two groups, one crossing from the positive and the other from the negative side of the plane. Considered in opposite directions, the mean characteristics (the number, mass, momentum, or energy) of these two groups are not necessarily equal. They may differ in consequence of the motion of the gas, the motion of the plane through the gas, or a varying condition of the gas, and the determination of the effect of these causes, particularly the last, on the mass, momentum and energy that may be carried across by either or both groups constitutes the extension of the dynamical theory of gases.

In order to take account of the difference in the two groups it is assumed, and so far there is nothing new in the assumption itself, that the group of molecules which crosses the surface from either side will partake of the characteristics of the gas in the region from which the molecules which constitute the group have come. The first direct step in the investigation is the deduction from the foregoing assumption of two theorems (I and II), supposing that there are no external forces. Taking $\sigma'(Q)$ to be the approximate value of $\sigma(Q)$ obtained on the assumption that the gas is uniformly in the mean condition which holds at the point xyz, the theorems I and II admit of the following symbolical expression:—

$$\sigma(Q)=\sigma'(Q)-s\left\{\frac{d}{dx}\sigma'(Q)+\frac{d}{dy}\sigma'(Q)+\frac{d}{dz}\sigma'(Q)\right\}.$$

Where s represents a certain distance, measured from the plane of reference.

This distance, s, enters as a quantity of primary importance into all the results of the investigation.

It is proposed to call s the mean range of the quantity Q, so as to distinguish it from the mean path of a molecule.

s is a function of the mean path, but it also depends on the nature of the impacts between the molecules. It is subsequently shown that—

$$s=\sqrt{\pi}\,a\mu,$$

from which it appears that $s \propto \frac{a}{\rho}$.

The dynamical conditions of steady density, steady momentum, and steady energy are then considered.

Putting $\Sigma(Q)$ for the value of Q in a unit of volume, in order that $\Sigma(Q)$ may be steady, we have—

$$\frac{d}{dx}\sigma_x(Q)+\frac{d}{dy}\sigma_y(Q)+\frac{d}{dz}\sigma_z(Q)=0,$$

whence, by giving to Q the value M, the mass of a molecule, we have the condition of steady density, for steady momentum Q has severally the values Mu, Mv, Mw, and for steady energy $Q=M(u^2+v^2+w^2)$.

The equations of motion are then applied to the particular cases which it is the object of this investigation to explain. Two cases are considered.

The first case is that of a gas in which the temperature and pressure vary only along a particular direction, so that the isothermal surfaces and the surfaces of equal pressure are parallel planes; this is *the case of transpiration.*

The second case is that in which the isothermal surfaces and surfaces of equal pressure are curved (whether of single or double curvature); this is the case of *impulsion* and the *radiometer.*

Transpiration.

As regards the first case, the condition of steady energy proved to be of no importance, but from the conditions of steady momentum and steady density, an equation is obtained between the velocity of the gas, the rate at which the temperature varies, and the rate at which the pressure varies, the coefficients being functions of the absolute temperature of the gas, the diameters of the apertures, and the ratio of these diameters to the mean range, which coefficients are known for the limiting conditions of the gas, *i.e.*, when the density is either very small or very large.

The most general form of this equation is—

$$\Omega=c\sqrt{\frac{2p}{\rho}}\left\{\mathrm{F}_1\left(\frac{c}{s}\right)\frac{1}{p}\frac{dp}{dx}-\mathrm{F}_2\left(\frac{c}{s}\right)\frac{\mathrm{M}}{\tau}\frac{d}{dx}\left(\frac{\tau}{\mathrm{M}}\right)\right\},$$

in which Ω is the mean velocity of transpiration along the tube, which is taken in the direction of the axis of x. M is the mass of a molecule, p the pressure of the gas, τ the absolute temperature, and c the semi-distance across the tube.

$$\mathrm{F}_1\left(\frac{c}{s}\right)=\frac{\sqrt{\pi}}{2}\left\{\mathrm{A}\,\frac{c}{s}+m\left[\lambda_1 f_1\left(\frac{c}{s}\right)+\lambda_2 f_2\left(\frac{c}{s}\right)\right]\right\},$$

$$\mathrm{F}_2\left(\frac{c}{s}\right)=\frac{1}{4\sqrt{\pi}}\left\{\pi m' f_3\left(\frac{c}{s}\right)+\frac{s}{c}\lambda_3 f_4\left(\frac{c}{s}\right)\right\}\left\{f\left(\frac{c}{s}\right)+\lambda_1 f_1\left(\frac{c}{s}\right)+\lambda_2 f_2\left(\frac{c}{s}\right)\right\}.$$

In which A, m, m' depend only on the shape of the section of the tube.

$f\left(\frac{c}{s}\right)$ is of the order $\frac{c}{s}$ when $\frac{c}{s}$ is zero, and is finite when $\frac{c}{s}$ is infinite,

$f_1\left(\frac{c}{s}\right)$ and $f_3\left(\frac{c}{s}\right)$ are unity when $\frac{c}{s}$ is zero, and zero when $\frac{c}{s}$ is infinite, and

$f_2\left(\frac{c}{s}\right)$ and $f_4\left(\frac{c}{s}\right)$ are zero when $\frac{c}{s}$ is zero, and unity when $\frac{c}{s}$ is infinite.

All these functions varying continuously between the limits here ascribed. Also—

λ_1 depends on the nature of the surface of the tube, but not upon the nature of the gas, while

λ_2 and λ_3 may depend both upon the gas and the surface.

From this equation, which is the general equation of transpiration, the experimental results, both with regard to thermal transpiration and transpiration under pressure, are deduced.

Impulsion.

In dealing with the second case, that in which the isothermal surfaces are curved, the three conditions—steady density, momentum, and energy—are all of them important.

These conditions reduce to an equation between the motions of the gas the variation in the absolute temperature and the variation in the pressure, with coefficients which involve the ratio of the mean range to the dimensions of the radii of curvature of the surfaces. The equation corresponds to the equation of transpiration, and as applied to the case in which heat is being conducted through a gas which is constrained to remain at rest, the equation becomes—

$$\frac{p-p_1}{p}=\frac{2}{\pi}s^2\frac{\mathrm{M}}{\tau}\frac{d^2}{dx^2}\left(\frac{\tau}{\mathrm{M}}\right),$$

or taking

$$s=\sqrt{\pi}\frac{a\mu}{2p},$$

$$p-p_1=\frac{\mu^2}{\rho}\frac{\mathrm{M}}{\tau}\frac{d^2}{dx^2}\left(\frac{\tau}{\mathrm{M}}\right),$$

$p-p_1$ being the excess of pressure in the direction of, and due to, the variation of temperature. In the abstract of a paper read in April last, Professor Maxwell gives an equation which, transformed into my symbols is—

$$p-p_1=\frac{3\mu^2}{\rho}\frac{\mathrm{M}}{\tau}\frac{d^2}{dx^2}\left(\frac{\tau}{\mathrm{M}}\right),$$

which only differs from mine in the coefficient — 3. As Professor Maxwell indicates that he has obtained his result without taking account of the tangential stresses, this difference is not a matter of surprise.

Besides the broad lines of the investigation which have been mentioned in this abstract, there are many minor points of which it is impossible to convey any adequate idea without going fully into the subject.

Some idea of the scope of the investigation may be gathered from the last section in the paper, which is accordingly introduced here.

Section XIII.—Summary and Conclusion.

Article 125. The several steps of the investigation which have been described may be enumerated as follows :—

(1.) The primary step from which all the rest may be said to follow is the method of obtaining the equations of motion so as to take into account not only the normal stresses which result from the mean motion of the molecules at a point, but also the normal and tangential stresses which result from a variation in the condition of the gas (assumed to be molecular). This method is given in Sections VI, VII, and VIII.

(2.) The method of adapting these equations to the case of transpiration through tubes and porous plates is given in Section IX. The equations of steady motion are reduced to a general equation expressing the relation between the rate of transpiration, the variation of pressure, the variation of temperature, the condition of the gas, and the lateral dimensions of the tube.

In Section X is shown the manner in which were revealed the probable existence (1) of the phenomena of *thermal transposition*, and (2) the law of correspondence between all the results of transpiration with different plates, so long as the density of the gas is inversely proportional to the linear lateral dimensions of the passages through the plates; from which revelations originated the idea of making the experiment on thermal transpiration and transpiration under pressure.

(3.) It is also shown in Section X that the phenomena of transpiration resulting from a variation in the molecular constitution of the gas (investigated by Graham) are also to be deduced from the equation of transpiration.

(4.) The method of adapting the equations of motion to the case of impulsion is given in Section XI.

In Section XII is shown how it first became apparent that the extremely low pressures at which alone the phenomena of the radiometer had been obtained were consequent on the comparatively large size of the vanes, and that by diminishing the size of the vanes similar results might be obtained at higher pressures, whence followed the idea of using the fibre of silk and the spider-line in place of the plate vanes.

(5.) In Section XII it is also shown that while the phenomena of the radiometer result from the communication of heat from a surface to a gas, as explained in my former paper, these phenomena also depend on the divergence of the lines of flow, whence it is shown that all the peculiar facts that have been observed may be explained.

(6.) Section II, Part I, contains a description of the experiments undertaken to verify the revelations of Section X respecting *thermal transpiration*, which experiments establish not only the existence of the phenomena, but also an exact correspondence between the results for the different plates at corresponding densities of the gas.

(7.) Section III contains a description of the experiments on *transpiration under pressure* undertaken to verify the revelations of Section X with respect to the correspondence between the results to be obtained with plates of different coarseness at certain corresponding densities of gas, which experiments proved not only the existence of this correspondence, but also that the ratio of the corresponding densities in these experiments is the same as the ratio of the corresponding densities with the same plates in the case of thermal transpiration—a fact which proves that the ratio depends entirely on the plates.

(8.) Section IV contains a description of the experiments with the fibre of silk, and with the spider-line undertaken to verify the revelations of Section XII, from which experiments it appears that, with these small surfaces, phenomena of impulsion, similar to those of the radiometer, occur at pressure but little less than that of the atmosphere.

Conclusion.

Article 126. As regards transpiration and impulsion, the investigation appears to be complete; most, if not all, the phenomena previously known have been shown to be such as must result from the tangential and normal stresses consequent on a varying condition of a molecularly constituted gas; while the previously unsuspected phenomena to which it was found that a variation in the condition of gas must give rise, have been found to exist.

The results of the investigation lead to certain general conclusions which lie outside the immediate object for which it was undertaken; the most important of these, namely, that gas is not a continuous plenum, has already been noticed in Article 5, Part I.

The Dimensional Properties of Gas.

Article 127. The experimental results considered by themselves bring to light the dependence of a class of phenomena on the relations between the density of the gas and the dimensions of the objects owing to the presence of which the phenomena occur. As long as the density of the gas is inversely proportional to the coarseness of the plates the transpiration results correspond; and in the same way, although not so fully investigated, corresponding phenomena of impulsion are obtained as long as the density of the gas in inversely proportional to the linear size of the objects exposed to its action;

in fact, the same correspondence is found with all the phenomena investigated.

We may examine this result in various ways, but in whichever way we look at it, it can have but one meaning. If in a gas we had to do with a continuous plenum, such that any portion must possess the same properties as the whole, we should only find the same properties, however small might be the quantity of gas operated upon. Hence, in the fact that we find properties of a gas depending on the size of the space in which it is enclosed, and on the quantity of gas enclosed in this space, we have proof that gas is not continuous, or, in other words, that gas possesses a dimensional structure.

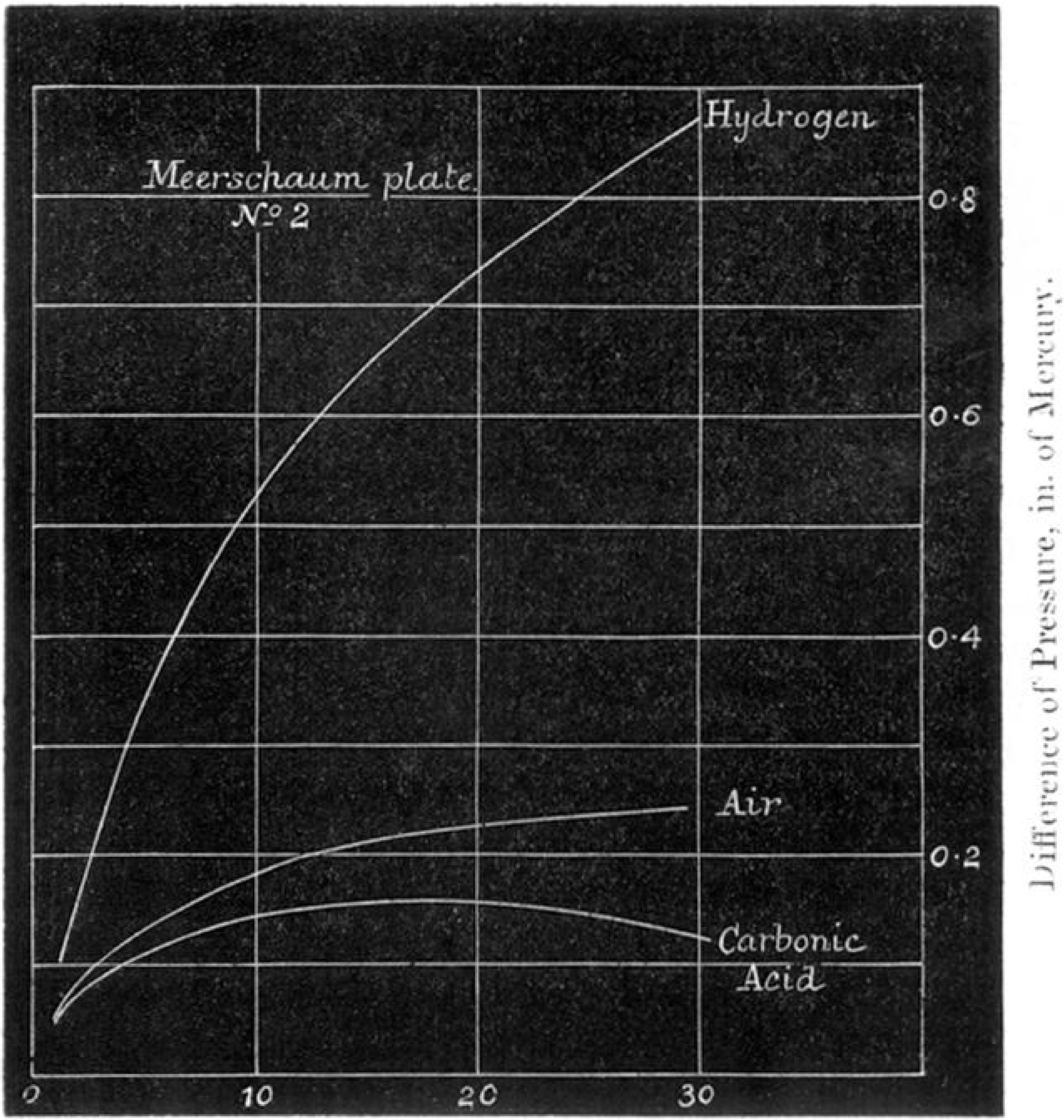
In virtue of their depending on this dimensional structure, and having afforded a proof thereof, I propose to call the general properties of a gas on which the phenomena of transpiration and impulsion depend, the *Dimensional Properties of Gas.*

This name is also indicative of the nature of these properties as deduced from the molecular theory; for by this it appears that these properties depend on the mean range, a linear quantity which, *cœteris paribus*, depends on the distance between the molecules.

In forming a conception of a molecular constitution of gas, there is no difficulty in realizing that there must exist such dimensional properties; there is, perhaps, greater difficulty in conceiving molecules so minute and so numerous that in the resulting phenomena all evidence of the individual action is lost; but the real difficulty is to conceive such a range of observational power as shall embrace, on the one hand, a sufficient number of molecules for their individualities to be entirely lost, while, on the other hand, it can be so far localized as regards time and space, that, if not the action of individuals, the action of certain groups of individuals, becomes distinguishable from the action of the entire mass. Yet this is what we have in the phenomena of transpiration and impulsion.

Although the results of the dimensional properties of gas are so minute that it has required our utmost powers to detect them, it does not follow that the actions which they reveal are of philosophical importance only; the actions only become considerable within extremely small spaces, but then the work of construction in the animal and vegetable worlds, and the work of destruction in the mineral world, are carried on within such spaces. The varying action of the sun must be to cause alternate inspiration and expiration, promoting continual change of air within the interstices of the soil as well as within the tissue of plants. What may be the effect of such changes we do not know, but the changes go on; and we may fairly assume that, in the processes of nature, the dimensional properties of gases play no unimportant part.

FIG. 1.



Mean Pressure in inches of Mercury.

FIG. 2.

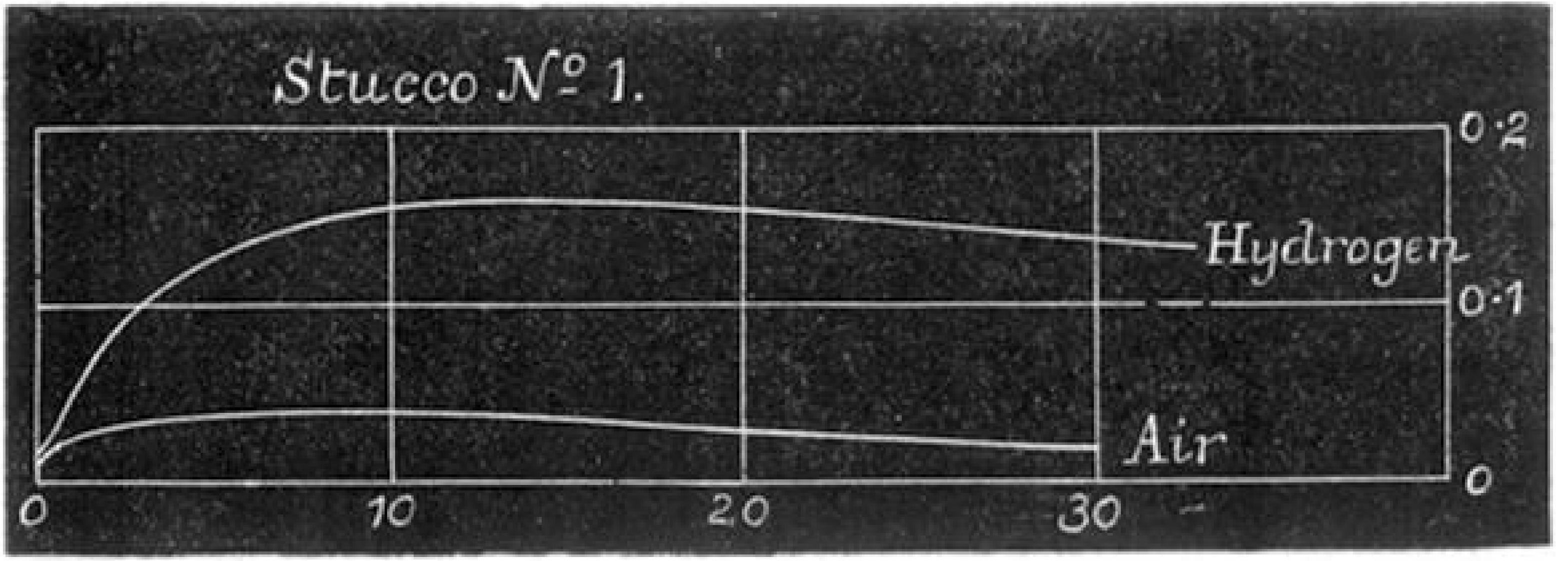
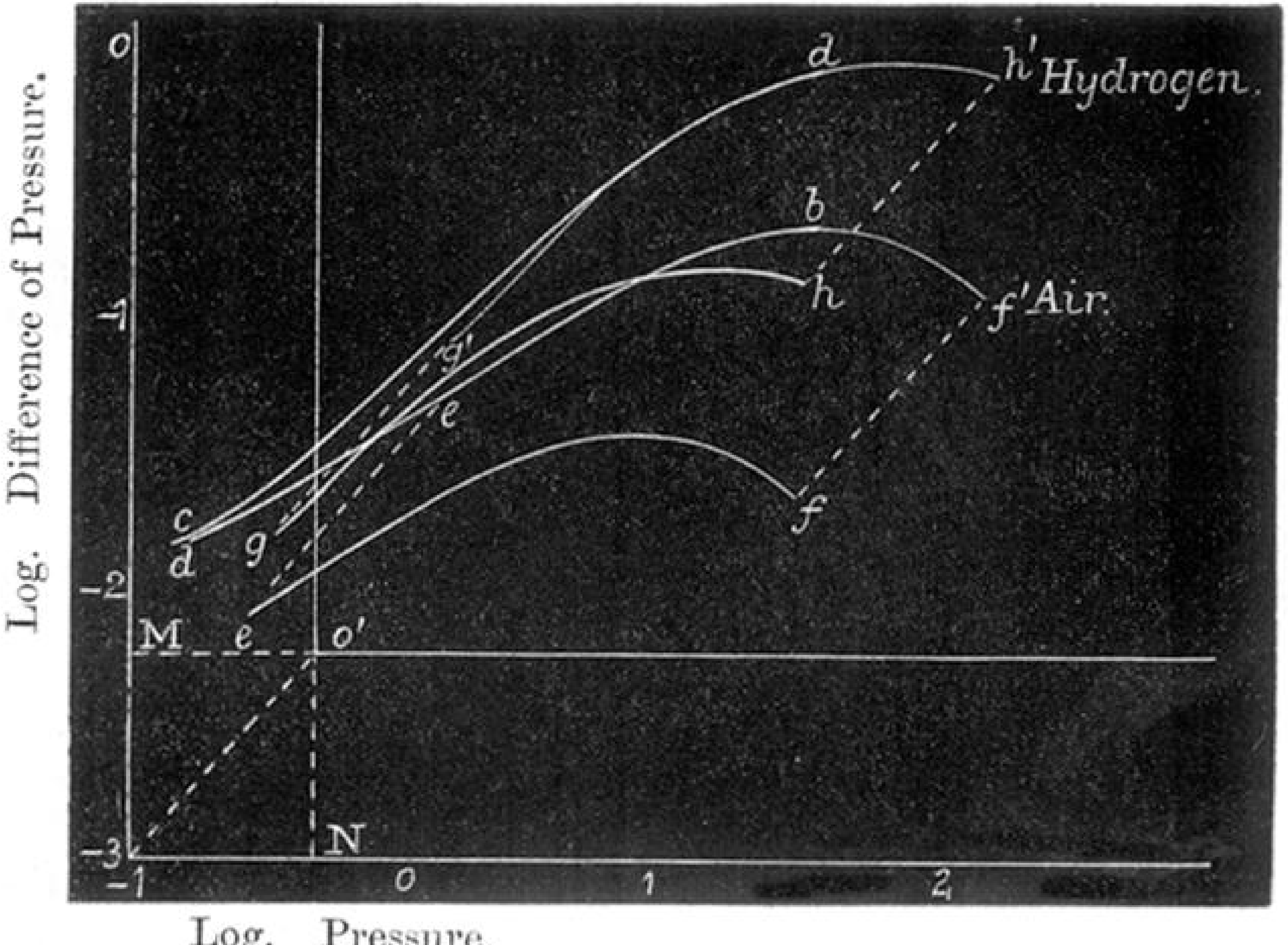


FIG. 3.



Log. Difference of Pressure.
0
-1
-2
-3
-1
0
1
2
a
h' Hydrogen.
b
h
f' Air.
g'
e
c
d
g
f
M
e
o'
N
Log. Pressure.